IDENTITA' VETTORIALI

Se $\mathbf{u} = u_x\,\hat{\mathbf{x}} + u_y\,\hat{\mathbf{y}} + u_z\,\hat{\mathbf{z}}$ allora (prodotto scalare) $\mathbf{u}\cdot\mathbf{v} = u_x v_x + u_y v_y + u_z v_z$

$\mathbf{v} = v_x\,\hat{\mathbf{x}} + v_y\,\hat{\mathbf{y}} + v_z\,\hat{\mathbf{z}}$

$\mathbf{w} = w_x\,\hat{\mathbf{x}} + w_y\,\hat{\mathbf{y}} + w_z\,\hat{\mathbf{z}}$

(prodotto vettoriale) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = (u_y v_z - u_z v_y)\,\hat{\mathbf{x}} + (u_z v_x - u_x v_z)\,\hat{\mathbf{y}} + (u_x v_y - u_y v_x)\,\hat{\mathbf{z}}$

modulo di $\mathbf{u}$ $= |\mathbf{u}| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{u_x^2 + u_y^2 + u_z^2}$

angolo compreso fra $\mathbf{u}$ e $\mathbf{v}$ $= \cos^{-1}\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$

identità dei prodotti tripli: $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u}) = \mathbf{w}\cdot(\mathbf{u}\times\mathbf{v})$ $\qquad \mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = (\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{u}\cdot\mathbf{v})\mathbf{w}$

IDENTITA' CONTENENTI GRADIENTE, DIVERGENZA, ROTORE E LAPLACIANO

$\nabla = \hat{\mathbf{x}}\dfrac{\partial}{\partial x} + \hat{\mathbf{y}}\dfrac{\partial}{\partial y} + \hat{\mathbf{z}}\dfrac{\partial}{\partial z}$ operatore "nabla" o "del"

$\nabla\phi(x,y,z) = \mathbf{grad}\,\phi(x,y,z) = \dfrac{\partial\phi}{\partial x}\,\hat{\mathbf{x}} + \dfrac{\partial\phi}{\partial y}\,\hat{\mathbf{y}} + \dfrac{\partial\phi}{\partial z}\,\hat{\mathbf{z}}$

$\mathbf{F}(x,y,z) = F_x(x,y,z)\,\hat{\mathbf{x}} + F_y(x,y,z)\,\hat{\mathbf{y}} + F_z(x,y,z)\,\hat{\mathbf{z}}$

$\nabla\cdot\mathbf{F}(x,y,z) = \mathbf{div}\,\mathbf{F}(x,y,z) = \dfrac{\partial F_x}{\partial x} + \dfrac{\partial F_y}{\partial y} + \dfrac{\partial F_z}{\partial z}$

$$\nabla\times\mathbf{F}(x,y,z) = \mathbf{rot}\,\mathbf{F}(x,y,z) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{z}}$$

$\mathbf{a}\cdot\nabla f = a_x\dfrac{\partial f}{\partial x} + a_y\dfrac{\partial f}{\partial y} + a_z\dfrac{\partial f}{\partial z}$

$(\mathbf{a}\cdot\nabla)\mathbf{F} = \left(\mathbf{a}\cdot\nabla F_x\right)\hat{\mathbf{x}} + \left(\mathbf{a}\cdot\nabla F_y\right)\hat{\mathbf{y}} + \left(\mathbf{a}\cdot\nabla F_z\right)\hat{\mathbf{z}}$

$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$\nabla\cdot(\mathbf{F}\times\mathbf{G}) = (\nabla\times\mathbf{F})\cdot\mathbf{G} - \mathbf{F}\cdot(\nabla\times\mathbf{G})$

$\nabla\cdot(\phi\mathbf{F}) = (\nabla\phi)\cdot\mathbf{F} + \phi\,(\nabla\cdot\mathbf{F})$

$\nabla\times(\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\cdot\mathbf{G}) - \mathbf{G}(\nabla\cdot\mathbf{F}) - (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F}$

$\nabla\times(\phi\mathbf{F}) = (\nabla\phi)\times\mathbf{F} + \phi\,(\nabla\times\mathbf{F})$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F}) + (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F}$

$\nabla\times(\nabla\phi) = 0$ $\quad$ (**rot grad** $= 0$)

$\nabla\cdot(\nabla\times\mathbf{F}) = 0$ $\quad$ (**div rot** $= 0$)

$\nabla^2\phi(x,y,z) = \nabla\cdot\nabla\phi(x,y,z) = \mathbf{div}\,\mathbf{grad}\,\phi = \dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2} + \dfrac{\partial^2\phi}{\partial z^2}$

$\nabla\times(\nabla\times\mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$ $\quad$ (**rot rot** $=$ **grad div** $-$ laplaciano)

VERSIONI DEL TEOREMA FONDAMENTALE DEL CALCOLO DIFFERENZIALE

$\displaystyle\int_a^b f'(t)\,dt = f(b) - f(a)$ $\quad$ (**teorema fondamentale** in una dimensione)

$\displaystyle\int_C \nabla\phi\cdot d\mathbf{r} = \phi\big(\mathbf{r}(b)\big) - \phi\big(\mathbf{r}(a)\big)$ se C è la curva $\mathbf{r} = \mathbf{r}(t)$, $(a \le t \le b)$

$\displaystyle\iint_R \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) dA = \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C \big(F_x(x,y)\,dx + F_y(x,y)\,dy\big)$ dove C è il contorno di R orientato positivamente $\quad$ (**teorema di Green**)

$\displaystyle\iint_S \nabla\times\mathbf{F}\cdot\hat{\mathbf{n}}\,dS = \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C \big(F_x(x,y,z)\,dx + F_y(x,y,z)\,dy + F_z(x,y,z)\,dz\big)$ dove C è il contorno orientato di S $\quad$ (**teorema di Stokes**)

Versioni tridimensionali: S è il contorno chiuso di V, con vettore normale esterno $\hat{\mathbf{n}}$

$\displaystyle\iiint_V \nabla\cdot\mathbf{F}\,dV = \oiint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$ $\qquad \displaystyle\int_V \nabla\cdot\mathbf{F} = \oint_S \mathbf{F}\cdot\hat{\mathbf{n}}$ $\qquad$ (**teorema della divergenza**)

$\displaystyle\iiint_V \nabla\phi\,dV = \oiint_S \phi\,\hat{\mathbf{n}}\,dS$ $\qquad \displaystyle\int_V \nabla\phi = \oint_S \phi\,\hat{\mathbf{n}}$ $\qquad$ (**teorema del gradiente**)

$\displaystyle\iiint_V \nabla\times\mathbf{F}\,dV = -\oiint_S \mathbf{F}\times\hat{\mathbf{n}}\,dS$ $\qquad \displaystyle\int_V \nabla\times\mathbf{F} = -\oint_S \mathbf{F}\times\hat{\mathbf{n}}$ $\qquad$ (**teorema del rotore**)