COORDINATE POLARI PIANE

vettore posizione: $\mathbf{r} = r\cos\theta\,\hat{\mathbf{x}} + r\sin\theta\,\hat{\mathbf{y}}$

fattori di scala: $h_r = \left|\frac{\partial \mathbf{r}}{\partial r}\right| = 1, \quad h_\theta = \left|\frac{\partial \mathbf{r}}{\partial \theta}\right| = r$

trasformazione: $r = \sqrt{x^2+y^2}$ $\quad x = r\cos\theta$

$\theta = \tan2^{-1}(y,x)$ $\quad y = r\sin\theta$

base locale: $\hat{\mathbf{r}}(\theta) = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$ $\quad \hat{\mathbf{x}} = \cos\theta\,\hat{\mathbf{r}}(\theta) - \sin\theta\,\hat{\boldsymbol{\theta}}(\theta)$

$\hat{\boldsymbol{\theta}}(\theta) = -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{y}}$ $\quad \hat{\mathbf{y}} = \sin\theta\,\hat{\mathbf{r}}(\theta) + \cos\theta\,\hat{\boldsymbol{\theta}}(\theta)$

elemento di area: $dV = r\,dr\,d\theta$

campo scalare: $f(r,\theta)$

campo vettoriale: $\mathbf{F}(r,\theta) = F_r(r,\theta)\,\hat{\mathbf{r}}(\theta) + F_\theta(r,\theta)\,\hat{\boldsymbol{\theta}}(\theta)$

gradiente: $\nabla f = \frac{\partial f}{\partial r}\,\hat{\mathbf{r}}(\theta) + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}}(\theta)$

divergenza: $\nabla\cdot\mathbf{F} = \frac{1}{r}\frac{\partial}{\partial r}\big(rF_r\big) + \frac{1}{r}\frac{\partial F_\theta}{\partial\theta}$

rotore: $\nabla\times\mathbf{F} = \left[\frac{1}{r}\frac{\partial}{\partial r}\big(rF_\theta\big) - \frac{1}{r}\frac{\partial F_r}{\partial\theta}\right]\hat{\mathbf{z}}$

laplaciano: $\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}$

lapl. vett.:
$$\nabla^2\mathbf{F} = \left[\nabla^2 F_r - \frac{F_r}{r^2} - \frac{2}{r^2}\frac{\partial F_\theta}{\partial\theta}\right]\hat{\mathbf{r}}(\theta) + \left[\nabla^2 F_\theta - \frac{F_\theta}{r^2} + \frac{2}{r^2}\frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\theta}}(\theta)$$

advezione: $\mathbf{a}\cdot\nabla f = a_r\frac{\partial f}{\partial r} + \frac{a_\theta}{r}\frac{\partial f}{\partial\theta}$

advez.:
$$(\mathbf{a}\cdot\nabla)\mathbf{F} = \left[a_r\frac{\partial F_r}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_r}{\partial\theta} - F_\theta\right)\right]\hat{\mathbf{r}}(\theta) + \left[a_r\frac{\partial F_\theta}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_\theta}{\partial\theta} + F_r\right)\right]\hat{\boldsymbol{\theta}}(\theta)$$

COORDINATE CILINDRICHE

vettore posizione: $\mathbf{r} = R\cos\theta\,\hat{\mathbf{x}} + R\sin\theta\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$

fattori di scala: $h_R = \left|\frac{\partial \mathbf{r}}{\partial R}\right| = 1, \quad h_\theta = \left|\frac{\partial \mathbf{r}}{\partial \theta}\right| = R, \quad h_z = \left|\frac{\partial \mathbf{r}}{\partial z}\right| = 1$

trasformazione: $R = \sqrt{x^2+y^2}$ $\quad x = R\cos\theta$

$\theta = \tan2^{-1}(y,x)$ $\quad y = R\sin\theta$

$z = z$ $\quad z = z$

base locale: $\hat{\mathbf{R}}(\theta) = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$ $\quad \hat{\mathbf{x}} = \cos\theta\,\hat{\mathbf{R}}(\theta) - \sin\theta\,\hat{\boldsymbol{\theta}}(\theta)$

$\hat{\boldsymbol{\theta}}(\theta) = -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{y}}$ $\quad \hat{\mathbf{y}} = \sin\theta\,\hat{\mathbf{R}}(\theta) + \cos\theta\,\hat{\boldsymbol{\theta}}(\theta)$

elemento di volume: $dV = R\,dR\,d\theta\,dz$

campo scalare: $f(R,\theta,z)$

campo vett.: $\mathbf{F}(R,\theta,z) = F_R(R,\theta,z)\,\hat{\mathbf{R}}(\theta) + F_\theta(R,\theta,z)\,\hat{\boldsymbol{\theta}}(\theta) + F_z(R,\theta,z)\,\hat{\mathbf{z}}$

gradiente: $\nabla f = \frac{\partial f}{\partial R}\,\hat{\mathbf{R}}(\theta) + \frac{1}{R}\frac{\partial f}{\partial\theta}\,\hat{\boldsymbol{\theta}}(\theta) + \frac{\partial f}{\partial z}\,\hat{\mathbf{z}}$

divergenza: $\nabla\cdot\mathbf{F} = \frac{1}{R}\frac{\partial}{\partial R}\big(RF_R\big) + \frac{1}{R}\frac{\partial F_\theta}{\partial\theta} + \frac{\partial F_z}{\partial z}$

rotore:
$$\nabla\times\mathbf{F} = \left[\frac{1}{R}\frac{\partial F_z}{\partial\theta} - \frac{\partial F_\theta}{\partial z}\right]\hat{\mathbf{R}}(\theta) + \left[\frac{\partial F_R}{\partial z} - \frac{\partial F_z}{\partial R}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[\frac{1}{R}\frac{\partial}{\partial R}\big(RF_\theta\big) - \frac{1}{R}\frac{\partial F_R}{\partial\theta}\right]\hat{\mathbf{z}}$$

laplaciano: $\nabla^2 f = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial f}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2}$

lapl. vett.:
$$\nabla^2\mathbf{F} = \left[\nabla^2 F_R - \frac{F_R}{R^2} - \frac{2}{R^2}\frac{\partial F_\theta}{\partial\theta}\right]\hat{\mathbf{R}}(\theta) + \left[\nabla^2 F_\theta - \frac{F_\theta}{R^2} + \frac{2}{R^2}\frac{\partial F_R}{\partial\theta}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[\nabla^2 F_z\right]\hat{\mathbf{z}}$$

advezione: $\mathbf{a}\cdot\nabla f = a_R\frac{\partial f}{\partial R} + \frac{a_\theta}{R}\frac{\partial f}{\partial\theta} + a_z\frac{\partial f}{\partial z}$

advez.:
$$(\mathbf{a}\cdot\nabla)\mathbf{F} = \left[\mathbf{a}\cdot\nabla F_R - \frac{a_\theta F_\theta}{R}\right]\hat{\mathbf{R}}(\theta) + \left[\mathbf{a}\cdot\nabla F_\theta + \frac{a_\theta F_R}{R}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[\mathbf{a}\cdot\nabla F_z\right]\hat{\mathbf{z}}$$
$$= \left[a_R\frac{\partial F_R}{\partial R} + \frac{a_\theta}{R}\left(\frac{\partial F_R}{\partial\theta} - F_\theta\right) + a_z\frac{\partial F_R}{\partial z}\right]\hat{\mathbf{R}}(\theta) + \left[a_R\frac{\partial F_\theta}{\partial R} + \frac{a_\theta}{R}\left(\frac{\partial F_\theta}{\partial\theta} + F_R\right) + a_z\frac{\partial F_\theta}{\partial z}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[a_R\frac{\partial F_z}{\partial R} + \frac{a_\theta}{R}\frac{\partial F_z}{\partial\theta} + a_z\frac{\partial F_z}{\partial z}\right]\hat{\mathbf{z}}$$